# Absorptive Bandstop Filter With Prescribed Negative Group Delay and Bandwidth

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*Abstract***— An absorptive bandstop filter is proposed and synthesized with prescribed negative group delay (NGD) and negative group delay bandwidth (NBW). It is realized with resistorloaded coupled-line structures where the NGD can be controlled by the Q-factors of resonators. The resistor of the first section is determined for impedance matching, also providing a negative delay. A prototype is developed, following the proposed design** procedure, with  $\overline{NGD} = -6.5$  ns and  $\overline{NBW} = 105$  MHz obtained. **The proposed method is experimentally validated by the good agreement between the synthesized and measured results.**

*Index Terms***— Absorptive bandstop filter, coupled microstrip line, negative group delay (NGD), resistor loading.**

### I. INTRODUCTION

IN recent years, networks with negative group delay (NGD) have drawn much attention, due to their potential appli-<br>existencing in communication systems, such as footforward line N recent years, networks with negative group delay (NGD) cations in communication systems, such as feedforward linearization amplifiers, antenna array systems, phase shifters, and power dividers. The NGD phenomenon can be observed within a limited frequency band of signal attenuation through certain media where the amplitude envelope of a signal is transmitted in advance of the undergoing delay [1], [2]. The low-frequency NGD circuits are realized by using series or shunt *RLC* resonators. Then, the distributed transmission line resonators are utilized in the microwave regime [3]–[7]. The conventional NGD circuits suffer from high in-band attenuation, poor NGD or magnitude flatness, and narrow negative group delay bandwidth (NBW). Attempting to overcome some of these issues, different methods have been presented. In [4] and [5], cross-coupled topologies are utilized to reduce the attenuation. In [6], the transversal filter is synthesized to improve the NGD flatness. In [7], reflective bandstop filters (BSFs) are designed with enhanced NBW, where an additional impedance transformer is required to obtain good in-band matching. However, the impedance transformer changes the reference impedance of the NGD BSF, and decreases the NGD and NBW. Therefore, careful tuning should

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Fig. 1. (a) Coupled-line section of the proposed BSF filter; (b) its equivalent transmission line model; and (c) its approximated circuit model.

be further carried out for the matched filter to satisfy the design specifications.

In this paper, an absorptive BSF with prescribed NGD is proposed. It is built up with resistor-loaded coupled-line structures, and the parameters are synthesized directly. The resistors are used to control the resonator *Q*-factors and absorption. Thus, the changing of reference impedance and NGD, introduced by the transformer, is avoided, and good return loss (RL) is obtained.

# II. THEORY AND DESIGN EQUATION

Fig. 1(a) shows a coupled-line section of the proposed BSF, and its equivalent transmission line model is shown in Fig. 1(b). By making the Y-parameters of Fig. 1(a) and (b) equal to each other, the element values in Fig. 1(b) are obtained as

$$
Z_{Ui} = \frac{2Z_{ei} Z_{oi}}{Z_{ei} + Z_{oi}}, Z_{Ci} = \frac{(Z_{ei} - Z_{oi})^2}{2(Z_{ei} + Z_{oi})}, R_{Ai} = R_i \left(\frac{Z_{ei} - Z_{oi}}{Z_{ei} + Z_{oi}}\right)^2.
$$
\n(1)

The transmission line model can further be approximated by the circuit in Fig. 1(c) when the electrical length  $\theta_i = \pi/2$ . Here, the unit element is regarded as a *K*-invertor, and the resistor-loaded λ/4 shorted stub is regarded as a lossy parallel resonant circuit. The susceptance slope parameter and the unloaded *Q*-factor of the resonator are derived as

$$
b_i = \pi/4 Z_{Ci}, \quad Q_i = b_i R_{Ai} = \pi R_i / (2 Z_{ei} + 2 Z_{oi}). \quad (2)
$$

Based on the above analysis, the proposed BSF in Fig. 2(a) can be represented by the circuit model in Fig. 2(b). It is a traditional BSF when the resistors are removed. So when setting all the inverter impedances to  $K_i = Z_{Ui} = Z_0$  except the first one,  $(3)$  can be obtained, similar to that in  $[8]$ 

$$
Z_{U1} = Z_0 \sqrt{m_1/(g_0 g_1)}, \quad Z_{Ci} = \pi Z_0 \text{FBW} m_i/4
$$
  
\n
$$
m_N = g_N g_{N+1} \text{ and } m_i = g_i g_{i+1}/m_{i+1} \text{ for } i = 1 \cdots N - 1
$$
\n(3)

where FBW is the fractional bandwidth, and  $g_i$  is the element value of a low-pass prototype. Then, the even- and odd-mode impedances are calculated from (1) as

$$
Z_{oi} = \frac{Z_{Ui}}{1 + \sqrt{Z_{Ci}/(Z_{Ci} + Z_{Ui})}}, Z_{ei} = \frac{Z_{Ui}}{1 - \sqrt{Z_{Ci}/(Z_{Ci} + Z_{Ui})}}.
$$
\n(4)

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Fig. 2. (a) Proposed absorptive BSF topology and (b) its equivalent circuit model.



Fig. 3. Layout of the fabricated BSF with NGD. Geometrical parameters are  $L_1 = L_3 = 17.8$ ,  $W_1 = W_3 = 1.55$ ,  $g_1 = g_3 = 0.32$ ,  $L_{p1} = 3.5$ ,  $L_{r11} = 6.23, L_{r12} = 3.8, L_2 = 18, W_2 = 1.41, g_2 = 0.22, L_{p2} = 3,$  $L_{r21} = 5.62, L_{r22} = 3.9, L_{p3} = 3.8, L_{r31} = 5.94, L_{r32} = 3.6$  (unit: mm). The resistances are  $R_{s1} = 110 \Omega$ ,  $R_{s2} = 620 \Omega$ , and  $R_{s3} = 680 \Omega$ .

When the *i*th resonator is referenced to the impedances  $Z_{Li}$ and  $Z_{Ri}$  for its left and right sides, respectively, its related transmission coefficient  $S_{21i}$  can be obtained as

$$
S_{21i} = \frac{2\sqrt{Z_{Li}Z_{Ri}}[R_{Ai}^{-1} + j(\omega C_i - \omega^{-1} L_i^{-1})]}{(Z_{Li} + Z_{Ri})[R_{Ai}^{-1} + j(\omega C_i - \omega^{-1} L_i^{-1})] + 1}.
$$
 (5)

Then, the NGD at the central frequency  $\omega_0$  is calculated by

$$
\tau_i = -\frac{d\angle S_{21i}}{d\omega}\bigg|_{\omega=\omega_0} = \frac{-2R_{Ai}^2b_i}{\omega_0(R_{Ai} + Z_{Li} + Z_{Ri})}.
$$
(6)

From (1) and (6), the relation between  $R_i$  and  $\tau_i$  is derived by

$$
R_i = -\frac{\omega_0 \tau_i}{4b_i} \left( 1 + \sqrt{1 - \frac{8(Z_{Li} + Z_{Ri})b_i}{\omega_0 \tau_i}} \right) \left( \frac{Z_{ei} + Z_{oi}}{Z_{ei} - Z_{oi}} \right)^2.
$$
 (7)

The first resistor is designed to match the lossy filter with the input port when resonating, and it can be derived as

$$
R_1 = R_{A1} \left( \frac{Z_{e1} + Z_{o1}}{Z_{e1} - Z_{o1}} \right)^2 = \frac{4 - Z_0 Z_{in1} (1/Z_{e1} + 1/Z_{o1})^2}{Z_0 (1/Z_{o1} - 1/Z_{e1})^2} \tag{8}
$$

where  $Z_{in1}$  is the input impedance of the rest part of filter seen from the right side of the first coupled-line section at  $\omega_0$ . Note that the parasitic capacitance of lumped resistors will shift the resonant frequencies, if the susceptance slope parameter of the resonator seen from the location of resistor is relatively small. In order to reduce the effect, the resistors are mounted close to the shorted ends of resonators, as shown in Fig. 3. *Rsi* is selected to make the *Q*-factor equal to the calculated one from (2), that is

$$
Q_i^{-1} \approx Q_{Rsi}^{-1} + Q_{i0}^{-1}, \ Q_{Rsi} \approx \pi R_{si} / [2(Z_{ei} + Z_{oi}) \sin^2 \theta_{si}]
$$
\n(9)

where  $\theta_{si}$  is the electrical length from the resistor to the shorted end of the *i*th resonator, and  $Q_{i0}$  is the  $Q$ -factor of *i*th resonator without loading resistors. Then, the resistance  $R_{si}$  is obtained as

$$
R_{si} \approx \frac{2(Z_{ei} + Z_{oi})\sin^2\theta_{si}}{\pi(1/Q_i - 1/Q_{i0})}, \ \theta_{si} \gg \arctan\frac{(Z_{ei} + Z_{oi})}{2R_i}.
$$
 (10)



Fig. 4. IL and required FBW versus: (a) prescribed NGD and the filter order *N* when NBW = 110 MHz and  $f_0 = 1.962$  GHz; (b) prescribed NBW/ $f_0$ and the filter order *N* when  $\tau_{total} = -6.5$  ns and  $f_0 = 1.962$  GHz.

The constraint of  $\theta_{si}$  in (10) is obtained by comparing the Y-parameters of the coupled-line sections in Figs. 1(a) and 3 at  $\omega_0$ . The responses of the filter will be slightly deteriorated with decreasing  $\theta_{si}$ . So  $\theta_{si}$  should be selected properly.

Set all delay  $\tau_i$  of the *i*th resonator ( $i = 2$  to *N*) equal to the predefined section delay  $\tau_C$  with  $Z_{Li} = Z_{Ri} = Z_0$  assumed, and the total delay of the proposed BSF is approximated by [7]

$$
\tau_{total} \approx a\tau_C + b \tag{11}
$$

where *a* is the correction factor to take into account the fact that the reference impedances of resonators working in the filter are different from  $Z_0$ , leading to the individual group delay  $\tau_i$  different from  $\tau_c$ , and *b* is a constant value to count for the delay introduced by the first resonator and the *K*-invertors. The value of *a* is mainly determined by the filter order *N* and FBW, and it can be obtained through the curvefitting method as

$$
a \approx (N - P_1) \left( \frac{P_2}{P_3 + \text{FBW}} + P_4 \right) \tag{12}
$$

where  $P_1 = 0.964$ ,  $P_2 = 0.024$ ,  $P_3 = 0.039$ , and  $P_4 = 0.97$ are obtained for the Chebyshev filter. Note that  $b = 0$  is used as an initial value since *b* is much smaller than  $\tau_{total}$  and the exact value of *a* and *b* will be updated during the subsequent design procedure.

The IL and required FBW are shown in Fig. 4(a) as the functions of prescribed NGD, with  $f_0 = 1.962$  GHz, NBW = 110 MHz, and  $N = 2, 3$ , and 4. They are also shown in Fig. 4(b) versus NBW/ $f_0$  when  $\tau_{total} = -6.5$  ns. It is seen that when NGD or NBW increases, both IL and FBW increase. So, there is a tradeoff among NGD, IL, and NBW. The IL increases with increasing *N*, and hence, lower order filters are preferred but with higher FBW required, which may lead to unrealizable filter topology, especially for  $N = 2$ .

Compared to the unmatched third-order NGD BSF, i.e.,  $R_1$  are also from (6), the NBW of the matched filter are almost the same. However, the 60-MHz NBW of matched filter in [7] is much smaller than that of the unmatched one, as shown in Fig. 5(a), where the resistances are tuned to achieve the same NGD of  $-6.5$  ns. The proposed matched filter provides the IL of 12.2 dB, 8 dB lower than that in [7]. The worst RL within the NGD band is improved by 20 dB, as shown in Fig. 5(b).

For the prescribed *f*0, τ*total*, and NBW, the design procedure is summarized in the following steps.

- *Step 1:* Initialize FBW = 0.1,  $N = 2$ , *a* with (12),  $b = 0$ and the *gi* of the corresponding low-pass prototype.
- *Step 2:* Calculate  $Z_{oi}$  and  $Z_{ei}$  with (4),  $\tau_{C1}$  with (11) and  $R_i$  with (7) and (8). Then, the first test value  $\tau_{total1}$ of NGD is obtained. Set  $\tau_{C2} = 0.9\tau_{C1}$  to obtain the second test value τ*total*<sup>2</sup> of NGD.



Fig. 5. Synthesized responses of the matched and unmatched third-order absorptive BSFs, compared to the filters in [7]. (a) Group delay and (b) magnitude, where the dashed lines with circles are for the matched cases, while the solid lines are for the unmatched cases.



Fig. 6. Measured, simulated, and modified results of the proposed filter.

- *Step 3:* Update *a* and *b* by solving (11) with the two tests  $[\tau_{C1}, \tau_{total1}]$  and  $[\tau_{C2}, \tau_{total2}]$ , and then obtain  $\tau_C$  from (11).
- *Step 4:* Update  $R_i$  with (7) and (8), and obtain the achieved NBW. If the NBW is not equal to the prescribed one, scale FBW and go to Step 2.
- *Step 5:* If the final calculated FBW is unrealizable, go to Step 1 with  $N = N + 1$ .
- *Step 6:* Calculate  $Q_i$  with (2), set proper values of  $\theta_{si}$  and  $Q_{i0}$ , and calculate  $R_{si}$  with (10).
- *Step 7:* Optimize the filter structure.

## III. RESULTS AND DISCUSSION

A third-order BSF prototype is designed, with the prescribed  $f_0 = 1.962$  GHz, NBW = 110 MHz, and  $\tau_{total} = -6.5$  ns. The element values of low-pass prototype are  $[g_0 \dots g_4]$  = [1, 0.6292, 0.9703, 0.6292, 1]. The values of  $\tau_C = -2.545$  ns, FBW = 13.5%,  $[Z_{o1}, Z_{o2}, Z_{o3}] = [40.0, 38.3, 40.0]$   $\Omega$ ,  $[Z_{e1}, Z_{e2}, Z_{e3}] = [66.67, 71.99, 66.67]$   $\Omega$ , and  $[R_1, R_2, R_3] =$ [576.9, 1768.8, 1942.1]  $\Omega$  are obtained by applying the above design steps. The desired *Q*-factor of each resonator is calculated by (2) to be  $[Q_1, Q_2, Q_3] = [8.49, 25.19, 28.60].$ The prototype is fabricated on a 1-mm-thick FR4 substrate, with  $\varepsilon_r = 4.4$  and tan $\delta = 0.02$ , where the *Q*-factors of  $\lambda$ /4 resonators without resistor loading are about  $Q_{i0} = 50$ . Here, we choose  $[R_{s1}, R_{s2}, R_{s3}] = [110, 620, 680] \Omega$ , which are available in practice, and  $\theta_{si}$  is about 0.42 rad from (10). To compensate for the difference between the odd- and even-mode electrical lengths of coupled microstrip line, the microstrip lines are decoupled near the shorted end [8]. The optimized structure and parameters are given in Fig. 3.

The simulated and measured S-parameters and NGDs of the prototype are shown in Fig. 6. Reasonable agreement is achieved between them. The measured response is centered at 1.99 GHz, and the frequency deviation of 28 MHz from the simulated one is mainly due to the tolerance of permittivity. The difference of RL between the measured and simulated results is mainly caused by the tolerances of resistor mounting and fabrication, especially the small gaps in the coupled

TABLE I PERFORMANCE COMPARISON OF PROPOSED NGD BSF WITH PREVIOUS WORKS

	<b>Size</b> $\lambda_g \times \lambda_g$	$f_{0}$ GHz	<b>NGD</b> ns	IL dB	<b>RL</b> dB	<b>NBW</b> MHz	NGD- <b>NBW</b>
$[3]$	$\approx 0.5 \times 0.5$	2.14	$-7.9$	16.5		60	0.474
[4]	$0.50 \times 0.30$	2.14	$-6.3$	20.7	6	70	0.441
$\left[5\right]$	$0.37 \times 0.48$	2.14	$-1.0$	3.8	12	60	0.060
[6]	$0.78 \times 0.06$	1.0	$-1.5$	33	25	370	0.555
$[7]$	$0.63 \times 0.42$	1.96	$-6.5$	21.2	9	81	0.527
<b>Ours</b>	$0.76 \times 0.16$	1.99	$-6.5$	22.6	21	105	0.683
$S_{V}$		196	$-65$	21.8	20	110	0.715

 $\lambda_{g}$ : the guided wavelength of a 50- $\Omega$  microstrip line at fo; Syn: the synthesized results of the proposed NGD BSF, and RL: the worst RL within the NGD band.

lines. The simulated result with slightly modified substrate  $\varepsilon_r = 4.28$  and geometric parameters  $[L_{r12}, L_{r22}, L_{r32}, g_2] =$ [4.35, 3.7, 3.55, 0.33] mm is also shown in Fig. 6 for comparison.

The measured NGD is −6.5 ns, with the IL of 22.6 dB and the worst RL of 21 dB within the NGD band. Table I shows the performance comparison of the proposed BSF with the previous works. Our synthesized results agree well with the measured ones except that the IL is smaller as expected. Due to the first coupled-line section working as an absorptive matching network also with an NGD, the proposed filter shows improved RL (by 12 dB) and enhanced bandwidth (by 30%) for the NGD band, when the NGD is the same as that in [7]. The NGD-NBW product is better than the other circuits with comparable RL. Our circuit size is smaller than the previous ones except for [6].

### IV. CONCLUSION

An absorptive BSF, based on coupled-line structures, has been designed with prescribed NGD and NBW. A set of resistors is loaded on the resonators so as to tune the *Q-*factors and match the BSF within its NGD band. The proposed method can be applied directly without further tuning the critical parameters in design. Good agreement is obtained between our simulated and measured results of the third-order filter prototype to demonstrate the proposed idea.

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