Novel Surface Impedance Modeling for Broadband Parameter Extraction of 3-D Interconnects

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***Abstract*— A novel boundary integral equation based method for modeling 3-D interconnects is proposed to compute the equivalent surface impedance in this letter. Differing from the traditional algorithm to discretize the conductor cross section, the proposed method only discretizes the contour of the conductor cross section, thus reducing the CPU time and memory cost. The loss characteristics of conductors describing the skin effect are taken into account through Green's function. The equivalent surface impedance model can be used to simplify electromagnetic simulation only using electric field integral equation. Numerical results show that the proposed method is both efficient and accurate in a broadband frequency, which is suited for modeling of 3-D interconnects and integrated passive structures.**

***Index Terms*— Boundary integral equation, electric field integral equation (EFIE), equivalent surface impedance.**

I. Introduction

As THE semiconductor industry moves to advanced process nodes, interconnect structures become more complicated with multiple thick metal layers embedded in inhomogeneous dielectric materials. Accurate and efficient broadband electromagnetic modeling of these interconnects is critical to ensure silicon success [1].

Numerical methods have been developed to extract the electrical model for interconnects within the semiconductor process. The per unit length RLGC parameters are commonly extracted in 2-D [2], even the lossy and semi-conductor substrates are taken into account [3]. General 3-D interconnect structures require electromagnetic solution in three dimensions with numerical methods such as the Finite Element Method (FEM) and the Method of Moments (MoM). MoM solution has gained in popularity because the mesh discretization is only performed on conductors while the complicated process profile approximated by dielectric layers is handled via the Green's function, which results in more efficient solution than that from the FEM. The conductor loss is typically represented with the surface impedance in the surface integral equation. The one-sheet model with the assumption of zero thickness conductor is used for conductors with high width-to-thickness ratio. The two-sheet model [4] is then developed to improve the accuracy as the width-to-thickness ratio becomes smaller. Generalized impedance boundary condition [5] as one type of equivalent principal based algorithm, is accurate for arbitrarily shaped conductor, however it is difficult to implement in multilayered media because of the magnetic field kernel. In [6], the 3-D volumetric formulations were simplified to surface based formulations. The equivalent surface impedance (ESI) model to correlate electric field and surface current was first proposed in [7], [8]. The properties of conductors are all comprised in the equivalent surface impedance.

In this letter, a novel method to obtain the equivalent surface impedance is proposed, which is based on the boundary integral equation. Numerical results demonstrate that the proposed method is both accurate and efficient, and is readily combined with EFIE for parameter extraction of 3-D interconnects.

II. Boundary Integral Equations

The boundary condition for the electric field on the surface of non-perfect 3-D conductors is

$$\hat{\boldsymbol{n}} \times (\boldsymbol{E_i} + \boldsymbol{E_s}) = \hat{\boldsymbol{n}} \times Z_s \boldsymbol{J_s} \tag{1}$$

Z_s represents the proportionality factor of the total electric field to the surface current [9]. In radiation and scattering problems, when the conductor skin depth is much smaller than the thickness of conductors, the Leontovich impedance boundary condition is capable. For small width-to-thickness ratio cases, in which the sidewall effect cannot be ignored, the surface impedance can be extended to the equivalent surface impedance on each side and obtained numerically [8].

The sectional dimensions of interconnects are usually in the scale of several to tens of microns, which are rather small compared with the wavelength of the operating frequency, typically below tens of GHz. Thus it is reasonable to assume that the electric current flows almost along the longitudinal direction, i.e., the signal propagation direction. For simplicity, the electric current $\boldsymbol{J}$ is assumed to be along the Z axis, so $\boldsymbol{J} = \hat{z} J_z$. According to the Helmholtz equation, the counterpart vector magnetic potential is $\boldsymbol{A} = \hat{z} A_z$.

In the interior region of conductor, A_z is governed by

$$\nabla^2 A_z(\boldsymbol{\rho}) - j\omega\mu_0\sigma A_z(\boldsymbol{\rho}) = \mu_0\sigma \frac{\mathrm{d}\phi}{\mathrm{d}z}, \quad \boldsymbol{\rho} \in S \tag{2}$$

in which ω is the angular frequency, σ is the conductivity, μ_0 is the permittivity of nonmagnetic material, S is the conductor cross section, ϕ is the scalar electric potential that satisfies the Coulomb Gauge. The right side of (2) can be treated as the excitation, and denoted as $V = \mathrm{d}\phi/\mathrm{d}z$.

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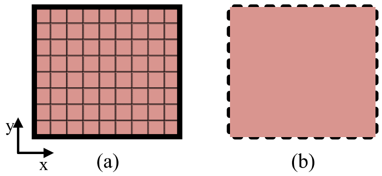

Fig. 1. An example of the cross section of interconnects: (a) discretization of cross section, (b) discretization of the contour.

According to Green formula

$$A_z(\boldsymbol{\rho}) = \oint_l \left[G_{in}(\boldsymbol{\rho}, \boldsymbol{\rho}') \frac{\partial A_z(\boldsymbol{\rho}')}{\partial n} - A_z(\boldsymbol{\rho}') \frac{\partial G_{in}(\boldsymbol{\rho}, \boldsymbol{\rho}')}{\partial n} \right] \mathrm{d}l' + j\frac{V}{\omega}\left[1 + \oint_l \frac{\partial G_{in}(\boldsymbol{\rho}, \boldsymbol{\rho}')}{\partial n} \mathrm{d}l'\right], \quad \boldsymbol{\rho} \in S \tag{3}$$

$G_{in}(\boldsymbol{\rho}, \boldsymbol{\rho}') = -\frac{j}{4} H_0^{(2)}(k|\boldsymbol{\rho} - \boldsymbol{\rho}'|)$ is the Green's function of 2-D Helmholtz equation, $H_0^{(2)}$ is the second-kind Hankel function of zeroth order, $k = \sqrt{-j\omega\mu_0\sigma}$ is the wavenumber in conductor material, $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ are the observation and source points respectively, l is the contour of S, and $\hat{n}$ represents the external normal direction.

In the exterior region of metal, A_z is governed by

$$\nabla^2 A_z(\boldsymbol{\rho}) = 0, \quad \boldsymbol{\rho} \in S^c \tag{4}$$

Still based on Green formula, we have

$$A_z(\boldsymbol{\rho}) = -\oint_l \Big[G_{ex}(\boldsymbol{\rho}, \boldsymbol{\rho}') \frac{\partial A_z(\boldsymbol{\rho}')}{\partial n} - A_z(\boldsymbol{\rho}') \frac{\partial G_{ex}(\boldsymbol{\rho}, \boldsymbol{\rho}')}{\partial n} \Big] \mathrm{d}l', \boldsymbol{\rho} \in S^c \tag{5}$$

$G_{ex}(\boldsymbol{\rho}, \boldsymbol{\rho}') = -\frac{1}{2\pi} \ln(|\boldsymbol{\rho} - \boldsymbol{\rho}'|)$ is the Green's function of 2-D Laplace equation, and S^c represents the non-conductor region. Be aware that the minus sign at the front of (5) denotes the internal normal direction.

The tangential electric field and equivalent surface current are expressed as

$$E_z(\boldsymbol{\rho}) = -j\omega A_z(\boldsymbol{\rho}) - V \tag{6}$$

$$J_s(\boldsymbol{\rho}) = -\frac{1}{\mu_0} \frac{\partial A_z(\boldsymbol{\rho})}{\partial n} \tag{7}$$

Differing from solving volume current distribution by discretizing cross section in Fig. 1(a), which may lead to irregular integral domain in 2-D, procedures for boundary integral equation method are versatile and can be uniformly implemented by only discretizing the contour of the cross section of arbitrary shape, which is capable to consider the influence of real process deviation.

The equivalent surface impedance is derived by

$$Z_s^{ESI}(\boldsymbol{\rho}) = \frac{E_z(\boldsymbol{\rho})}{J_s(\boldsymbol{\rho})} \tag{8}$$

which extends the method to derive the Leontovich impedance boundary condition directly.

III. Implementation of Method of Moments

Compared with the discretization model in [8], the proposed boundary integral equation based method only discretizes the contour, thus reducing the number of unknowns apparently. The simultaneous equations of (3) and (5) can be solved via MoM procedures. The A_z and $\partial A_z/\partial n$ are expanded using piece-wise basis functions

$$A_z(\boldsymbol{\rho}) = \sum_{i=1}^{N} \alpha_i g_i(\boldsymbol{\rho}), \quad \frac{\partial A_z}{\partial n} = \sum_{i=1}^{N} \beta_i g_i(\boldsymbol{\rho}) \tag{9}$$

$$g_i(\boldsymbol{\rho}) = \begin{cases} 1, & \boldsymbol{\rho} \in l_i \\ 0, & \boldsymbol{\rho} \notin l_i \end{cases} \tag{10}$$

N is the number of the discretized segments of the contour, l_i is the i-th segment. The $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_N]$ and $\boldsymbol{\beta} = [\beta_1, \cdots, \beta_N]$ are the coefficient vectors to be solved.

By eliminating the singular term and adopting point matching method in the test procedure, the resulting matrix equation is

$$\begin{bmatrix} \boldsymbol{T_1} & \boldsymbol{T_2} \\ \boldsymbol{T_3} & \boldsymbol{T_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{0} \end{bmatrix} \tag{11}$$

The entries of $\boldsymbol{T_i}(i = 1, \cdots, 4)$ and $\boldsymbol{U}$ are calculated by line integral in (3) and (5). Once the surface impedance is obtained through (8), we can substitute Z_s in (1) with Z_s^{ESI} to solve two-region problems.

By representing the scattering electric field in (1) through integral formulation, the EFIE is

$$\hat{n} \times \left[E_i(r) - j\omega\mu_0 \int_S \bar{G}(r, r') \cdot J_s(r') \mathrm{d}S'\right] = \hat{n} \times Z_s^{ESI}(r) J_s(r) \tag{12}$$

$\bar{G}(\boldsymbol{r}, \boldsymbol{r}')$ is the dyadic Green's function which depends on the properties of the substrates. The Rao-Wilton-Glisson (RWG) discretization and the Galerkin's method can be adopted to in the MoM solution to EFIE.

It should be noted that the ESI model is built based on the approximation that the surface impedance just relies on the conductor itself and the coupling between conductors and the substrate effects are captured in EFIE. Besides, the ESI does not take account for the high order mode at the corners of interconnects. However it has little effect on the accuracy of the current extraction in low order MoM schemes [6].

IV. Numerical Results

The equivalent surface impedance along the contour should be pre-computed before applying EFIE. The accuracy and the efficiency of the proposed method are illustrated in Fig. 2. The cross section of the interconnects is square with a side length of 40 μm. The conductivity is 3.57×10^7S/m, and the operating frequency is 10 GHz. The results from volume integral equation based method converge to those from boundary integral equation based method with mesh refinement, but require the size of the mesh should be no coarser than half the skin depth. Actually even with the same division size, the number of unknowns for the proposed method is much less.

Two numerical examples in 3-D are studied to validate the proposed method. In the first example, the resistance and

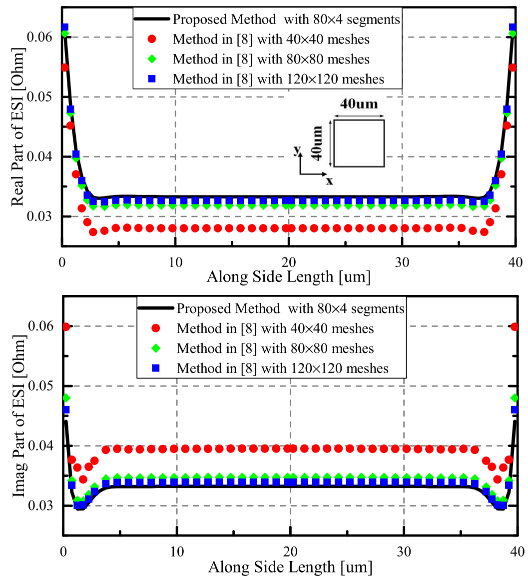

Fig. 2. Distribution of the equivalent surface impedance along the side of cross section.

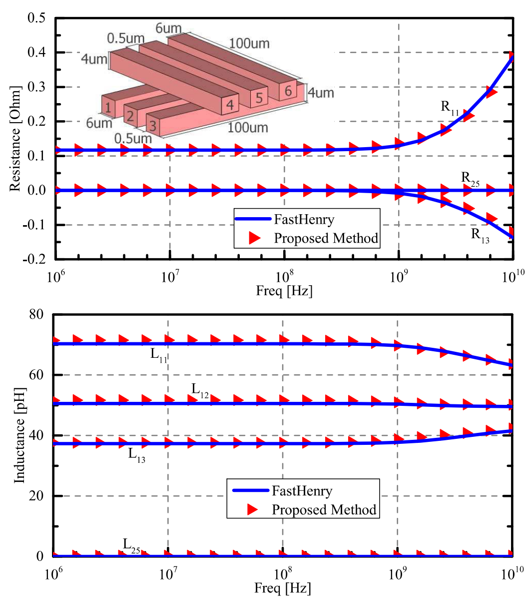

Fig. 3. Extracted resistance and inductance for 3×2 crossing interconnects. Sequence numbers and related geometric dimensions are labeled.

inductance of a 3×2 crossing interconnects are extracted. The cross section of the interconnects is rectangle with a width of 6 μm and a thickness of 4 μm, and the conductivity is 3.57×10^7S/m. Compared with the volume integral equation based method (FastHenry), the maximum error of the proposed method is less than 2%, even the spacing between parallel interconnects is 0.5 μm and the strong coupling effects arise, as is shown in Fig. 3. The proposed method is valid in many cases which the semiconductor process for integrated passives in IC is no higher than the level of sub micron.

The second numerical example is about an integrated spiral inductor with right-angled bends. The linetype and the material are the same as those in the first example. As is shown in Fig. 4, the quality factor and inductance of the inductor extracted from the proposed method are very close to those

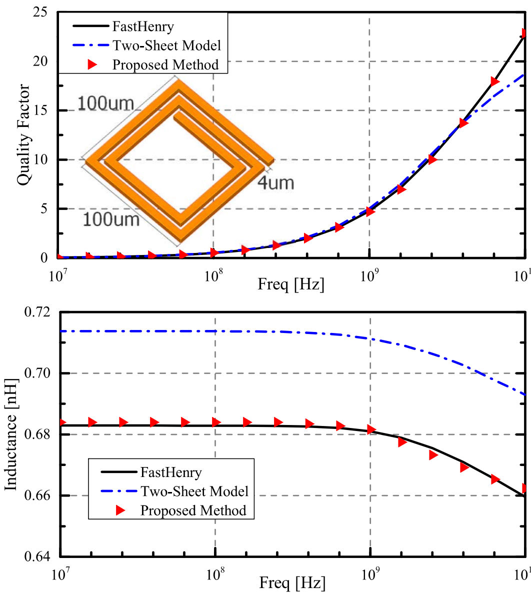

Fig. 4. Extracted Q factor and inductance for integrated spiral inductor.

from FastHenry with the maximum relative error less than 1%. While the two-sheet model [4] obviously shows larger deviations for ignoring the sidewall effect. The proposed method is still valid when corners do not account for most part of the layout.

V. Conclusion

A boundary integral equation based method is proposed to extract the equivalent surface impedance model. The proposed method is both accurate and efficient, which can be readily combined with EFIE for broadband parameter extraction of 3-D interconnects.

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